

## Degradation of IPU

Introduction

Laplace

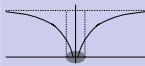
Heat Equation

Continuous IPU  
introduction

END

# Degradation of IPU

3. Mai 2013



## Degradation of IPU

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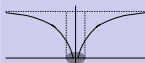
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① Introduction

② Laplace

③ Heat Equation

④ Continuous IPU introduction

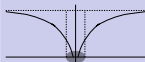


## Structure of the thesis:

2.5 chapters

- The limit  $t \rightarrow \infty$ . (Laplace Equation)  
Proof of concept and tools
- Introduce time. (Heat Equation)  
Fit parameters to match the experimental Data.
- Limit  $t \rightarrow \infty$  + nat. degradation proportional to depth.  
Simulate field conditions.

# Introduction



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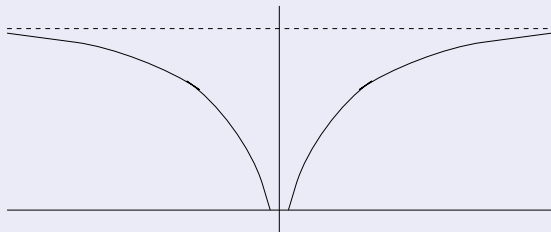
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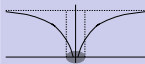
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Graphic's description - case  $t \rightarrow \infty$

- Constant IPU-Level + IPU-sink (seramis)
- Fundamental solution  $u(x) = \frac{-c}{x} + u_0$ .

# Introduction



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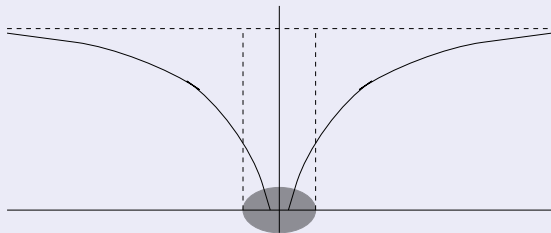
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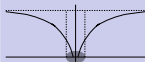
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## Graphic's description - the seramis

- no negative concentration outside seramis
- the system is controlled by the seramis' boundary.

# Introduction



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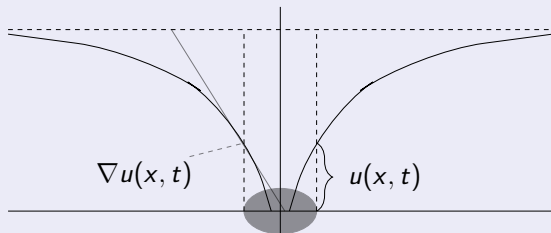
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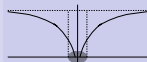
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## Mathematical essence

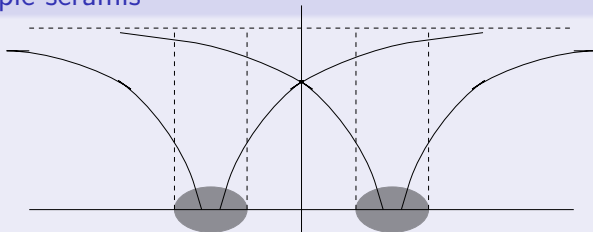
- Diffusion  $0 = D\Delta u$  (Laplace Equation).
- Boundary  $D\frac{du}{dr}|_{r=R} = \alpha u|_{r=R}$ .

# Laplace - Power series



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## Multiple seramis



## Problem

Seramis don't communicate yet.

Solution → Correction term determined by power series.

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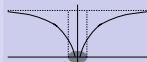
**Laplace**

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# Laplace - Power series



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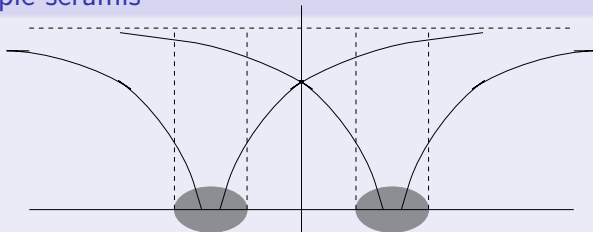
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## Multiple seramis



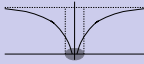
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# Laplace - Historical examples



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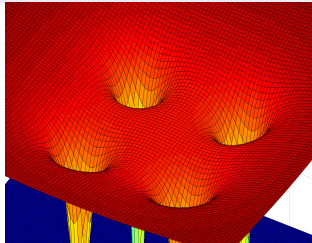
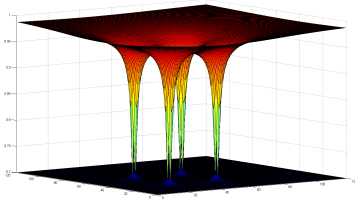
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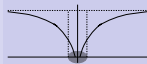
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Testplots made some time ago.



# Introduce Time - The Heat Equation



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## The Difference

- Laplace:  $0 = \Delta u$
- Heat Eq.:  $u_t = \Delta u$
- + natural degradation  $u_t = \Delta u - \gamma u$

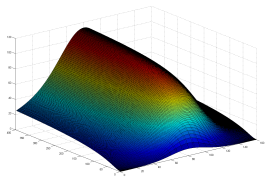
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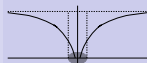
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# Introduce Time - The Heat Equation



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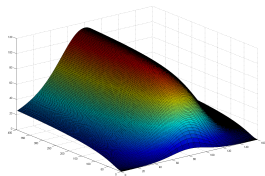
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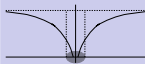
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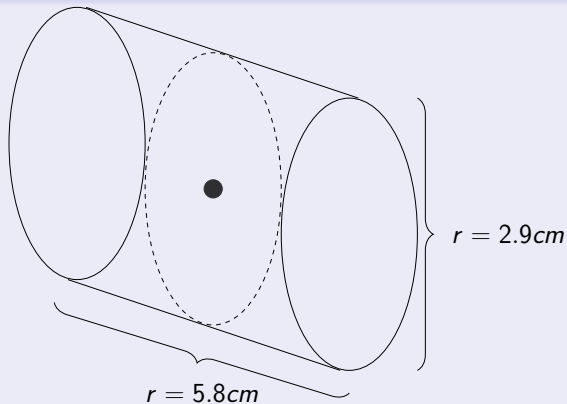
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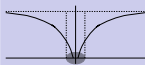
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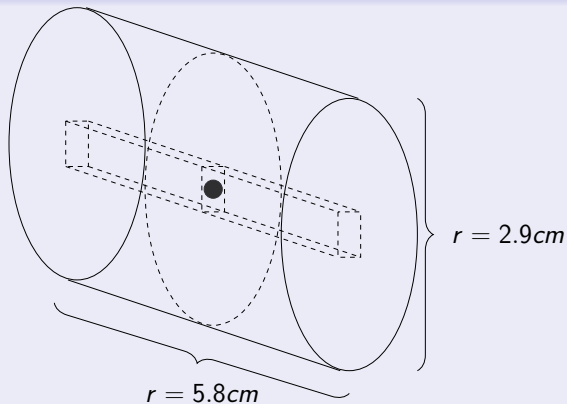


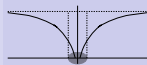
## Testtube and seramis



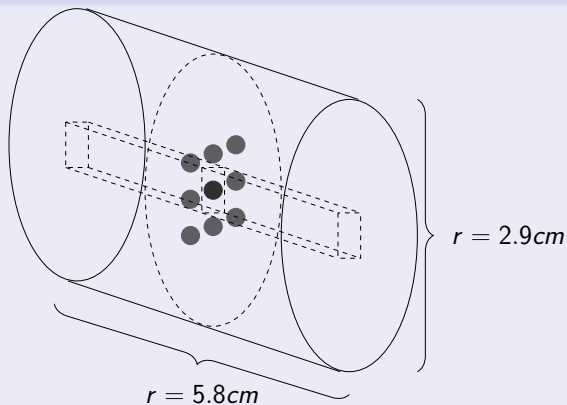


## Only examine the local environment

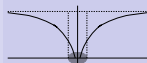




## Mirrored seramis to fulfill boundary conditions



# Continuous IPU Introduction



Degradation of  
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## Comparison to Laplace

- Same  $t \rightarrow \infty$
- But natural degradation

## Example: With $\leftrightarrow$ without seramis

$u_0 = 1, D = 0.021, \gamma = 0.03, a_0 = 0.15, R_0 = .2.$

Shown layers 0, -1, -1.5.

A test integration shows that in layer -3 the square  $(-0.5, -0.5)(0.5, 0.5)$  contains

- With Seramis:  $0.013394 \frac{\text{mol}}{\text{m}}.$
- Without Seramis:  $0.027718 \frac{\text{mol}}{\text{m}}.$

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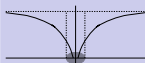
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# Continuous IPU Introduction



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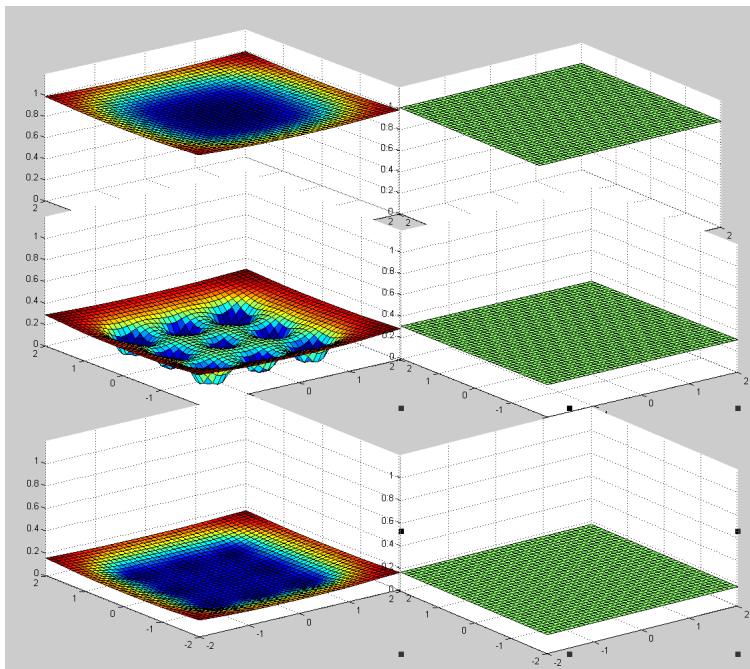
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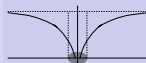
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# END



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# Questions?

Debug: You saw 11 frames!